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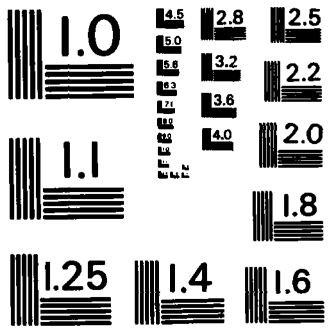

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ITEM #20, CONTINUED: homogeneous rest region of the body.

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(2)

ON WAVE AND ENTROPIC AMPLITUDES IN MAXWELLIAN MATERIALS†

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Abstract

Thermodynamic influences on the behavior of acceleration waves in a class of general Maxwellian non-conductors are examined. It is shown that every acceleration wave must be homentropic but that, in general, the entropic amplitude and the amplitude of the wave are not proportional as they would be in the special case of a simple non-conductor with fading memory; a necessary and sufficient condition for the proportionality of the two types of amplitudes is given. The Bernoulli equation which governs the growth behavior of the wave is derived and simple solutions are obtained for waves which propagate into a homogeneous rest region of the body.

Introduction

In his groundbreaking paper, Coleman [1] laid the foundations of a comprehensive thermo-dynamical theory for simple materials with fading memory; the theory established, was then used, by Coleman and Gurtin [2], to study the effects of thermodynamic influences on the growth behavior of acceleration waves propagating both in (simple) definite conductors of heat and in non-conductors. Coleman's initial work was followed by papers by Gurtin [3] and Wang & Bowen [4] in which attempts were made, respectively, to clarify the basic structure of Coleman's original theory and to propose an alternative thermodynamical theory for non-linear materials which retains, at least, some of the best features of Coleman's theory while avoiding some of its most striking drawbacks. Finally, in the last of a major series of papers on the propagation and growth behavior of waves in materials with memory, Coleman, Greenberg, and Gurtin [5] put forth a concrete constitutive proposal for the mechanical response of a Maxwellian material of class N ($N \geq 1$, an integer) and proved that every sufficiently smooth simple material with fading memory could be considered a Maxwellian material of order $N(N \geq 2)$ relative to a well-defined class of C^{N-1} motions; the growth behavior of acceleration waves (and higher order waves) in Maxwellian bodies was examined and the results were then applied to study the growth behavior of waves propagating both in smooth elastic materials and in sufficiently smooth simple materials with fading memory.

In this paper we extend the previous work on Maxwellians materials to take account of thermodynamic influences in the case where the material is a non-conductor of heat. After establishing certain kinematical and

thermodynamical preliminaries, we present our definition of a Maxwellian Non-Conductor of Class 1 in §1. In §3, thermodynamical restrictions on the response of a Maxwellian Non-Conductor, which are implied by the Clausius-Duhem inequality, are examined; in doing so we make one basic assumption on the underlying class of processes, which occurs in the definition of the material, and impose certain smoothness requirements on both the stress and temperature fields. Extending a result of Coleman & Gurtin [2], for waves in simple non-conductors with memory, it is shown that every such wave in a general Maxwellian Nonconductor must be homentropic. The intrinsic velocity of an acceleration wave, which is propagating in a Maxwellian Non-Conductor, is then calculated. Finally, we derive a differential equation which governs the growth behavior of acceleration waves propagating in Maxwellian Non-Conductors; in order to do this we find it necessary to relate the entropic amplitude, as defined by Coleman & Gurtin [2], to the amplitude of the wave. We find that, in a general Maxwellian Non-Conductor of Class 1, the <u>two amplitudes are not proportional and proportionality holds if and only if a simple explicit relationship connects two of the constitutive coefficients at the wave front</u>.

The differential equation which governs the growth behavior of acceleration waves in Maxwellian Non-Conductors of Class 1 is a Bernoulli equation whose coefficients are rather complicated functions involving not only the values of the various constitutive quantities and their derivatives at the wave but also the values of both the spatial and time derivatives of the deformation gradient and entropy ahead of the wave. Several simplifications occur when we consider acceleration waves which propagate into a homogeneous rest region of the body. The solutions, of the governing differential equation, which are

well-known from previous work on the growth behavior of acceleration waves in non-linear materials of various kinds, are recorded and analyzed. In doing this, we make explicit the dependence of the amplitude, of the wave, on the various thermodynamic entities which occur in the theory; <u>the previous results for acceleration waves propagating in general Maxwellian materials of class 1, in the purely mechanical theory, can not be deduced from ours by simply suppressing all of the thermodynamical variables</u>.

Mechanical and Thermodynamical Preliminaries

All the kinematical and mechanical concepts used in this paper are one dimensional. Let $\mathcal{R}$ denote a closed interval of the real line; a motion of the body is described by a function $x = \chi(X,t)$ which gives the location x at time t of the material point which has the position X in the reference configuration; the density in $\mathcal{R}$ is denoted by $\rho_{\mathcal{R}}$. Not only the motion χ, but also the stress T, the absolute temperature θ, the specific internal energy per unit mass ϵ, the specific entropy η, the heat flux q, the specific extrinsic body force b, and the specific extrinsic heat supply r are all to be regarded as functions of X and t; we assume that $\rho_{\mathcal{R}}$ is constant over B.

When they exist, the derivatives $\dot{x}(X,t) = \partial_t\chi(X,t)$, $\ddot{x}(X,t)$

$= \partial_t^2\chi(X,t)$ and $F(X,t) = \partial_X\chi(X,t)$ are called, respectively, the velocity, acceleration, and deformation gradient. The laws of balance of momentum and balance of energy take the forms

$$(1.1_1) \qquad \frac{d}{dt}\int_{X_\beta}^{X_\alpha} \dot{x}\rho_R dX = \int_{X_\beta}^{X_\alpha} b\rho_R dX + T(X_\beta,t) - T(X_\alpha,t)$$

$$(1.1_2) \qquad \frac{d}{dt}\int_{X_\beta}^{X_\alpha}\left(\frac{\dot{x}^2}{2} + \varepsilon\right)\rho_R dX = \int_{X_\beta}^{X_\alpha}(\dot{x}b + r)\rho_R dX + T(X_\beta,t)\dot{x}(X_\alpha,t)$$

$$- T(X_\alpha,t)\dot{x}(X_\alpha,t) - q(X_\beta,t) + q(X_\alpha,t)$$

and (1.1_1), (1.1_2) are assumed valid at all times t and for every pair of points X_α, X_β in R. An admissible thermdynamic process for the body is a specification of the fields x, θ, T, η, ε, q, b, r, as functions of X and t, in such a way as to be compatible with the balance laws (1.1_1), (1.1_2) and whatever constitutive relations are prescribed for the material. In the present work those constitutive assumptions take the following forms: Let J be an open subset of $(0,\infty)$ and let E be the set of all functions from $J\times J\times R\times(-\infty, t_o)$ into the real numbers. Then

<u>Definition 1.1</u> A material body is said to be a Maxwellian Non-Conductor of class 1 if $q\equiv 0$ and if there exists a non-empty class M consisting of pairs (χ,η), where χ is a C^1 motion on $R\times(-\infty,t_o)$ and η is a continuous entropy density function on $R\times(-\infty,t_o)$ such that the following holds (1) $\dot{T}$, $\dot{\varepsilon}$, and $\dot{\theta}$ exist whenever $\dot{F}$ and $\dot{\eta}$

do and (ii) corresponding to each $\xi = (\chi,\eta) \in \mathcal{M}$ there exist nine C^1 functions A_ξ, B_ξ, ..., I_ξ in $\mathcal{E}$ such that

$$(1.2_1) \qquad T(X,t) = A_\xi(F;\eta;X;t)\dot{F} + B_\xi(F;\eta;X;t)\dot{\eta} + C_\xi(F;\eta;X;t)$$

$$(1.2_2) \qquad \dot{\varepsilon}(X,t) = D_\xi(F;\eta;X;t)\dot{F} + E_\xi(F;\eta;X;t)\dot{\eta} + F_\xi(F;\eta;X;t)$$

$$(1.2_3) \qquad \dot{\theta}(X,t) = G_\xi(F;\eta;X;t)\dot{F} + H_\xi(F;\eta;X;t)\dot{\eta} + I_\xi(F;\eta;X;t)$$

Finally, we take as our expression for the second law of thermodynamics, the Clausius-Duhem inequality,

$$(1.3) \qquad \frac{d}{dt}\int_{X_\beta}^{X_\alpha} \eta\rho_R dX \geq \int_{X_\beta}^{X_\alpha} \frac{r}{\theta}\rho_R dX + \frac{q(X_\alpha,t)}{\theta(X_\alpha,t)} - \frac{q(X_\beta,t)}{\theta(X_\beta,t)}$$

which must hold at all times t and for every pair of points X_α, X_β in R. We will examine the consequences of requiring that (1.3) hold for all smooth admissible therodynamic processes in a Maxwellian Non-Conductor.

2. General Properties of Acceleration Waves

We collect here those facts about acceleration waves which are pertinent to a study of the propagation and growth behavior of such waves in Maxwellian Non-Conductors. The material representation of a wave is a smooth one-parameter family of points $Y(t) \in R$, $-\infty < t < t_o$, where $Y(t)$ gives the material point (labeled by its position in R) at which the wave is to be found at time t. The material trajectory Σ of the wave is given by

$$\sum = \{X,t)|X = Y(t),\ t\varepsilon(-\infty,t_o)\} \tag{2.1}$$

and we call the wave an acceleration wave if the fields $x(X,t)$ and $\eta(X,t)$ have the following properties: x, $\dot{x}$, F, and η are continuous functions of X and t jointly for all X and t, while $\ddot{x}$, $\dot{F}$, $\partial_X F$, $\dot{\eta}$, $\partial_X\eta$ and also x, $\ddot{F}$, $\partial_X\dot{F}$, $\partial_X^2 F$, $\ddot{\eta}$ $\partial_X\eta$ $\partial_X^2\eta$ have (at most) jump discontinuities across $\sum$ but are continuous in X and t jointly everywhere else. If $f(X,t)$, as a function of X, has only a jump discontinuity at $X = Y(t)$, then the jump in $f(X,t)$ across $\sum$ at time t is defined by

$$[f] = \lim_{X\to Y(t)^-} f(X,t) - \lim_{X\to Y(t)^+} f(X,t) = f(Y(t)^-,t) - f(Y(t)^+,t \tag{2.2}$$

whenever the intrinsic velocity, $U_t = \frac{d}{dt} Y(t)$, of the wave is positive, $f^- = f(Y(t)^-,t)$ and $f^+ = f(Y(t)^+,t)$ represent the limiting values of $f(X,t)$ immediately behind and just in front of the wave. By a well-known theorem of Maxwell we have the following compatability conditions for acceleration waves:

$$[\ddot{x}] = -U_t[\dot{F}] = U_t^2[\partial_X F] \tag{2.3}$$

$$[\dot{\eta}] = -U_t[\partial_X\eta] \tag{2.4}$$

Now let us assume that the extrinsic body force and the heat supply are assigned in such a way that $b(X,t)$, $r(X,t)$, $\dot{b}(X,t)$, and $\partial_X r(X,t)$ are continuous functions of X and t. Then for processes involving

acceleration waves, the laws of balance of momentum and energy (1.1_1) and (1.1_2) are together equivalent to the assertion that for $X \neq Y(t)$

$$(2.5_1) \qquad \partial_X T + \rho_R b = \rho_R \ddot{x}$$

$$(2.5_2) \qquad \rho_R \dot{\varepsilon} = T\dot{F} + \rho_R r$$

while for $X = Y(t)$

$$(2.6_1) \qquad [\partial_X T] = \rho_R [\ddot{x}]^{(1)}$$

$$(2.6_2) \qquad \rho_R [\dot{\varepsilon}] = [T\dot{F}]$$

In both (2.5_2) and (2.6_2) we have set $q = 0$ in accordance with our assumption that the material is a non-conductor.

3. Thermodynamical Restrictions on Maxwellian Non-Conductors

We discuss here restrictions, on the constitutive equations of a Maxwellian Non-Conductor, which are implied by the Clausius-Duhem inequality (1.3) when we make one additional assumption on the class of processes M and impose certain smoothness requirements on the stress and temperature fields.

---

(1) We are assuming that ρ_R is constant over R so that $\rho_R^t = \rho(X)|_{X=Y(t)} = \rho_R$; this assumption has the effect of eliminating a term involving $\frac{d}{dt}\rho_R^t = (\partial_X \rho)_t U_t$ in the equation which governs the the growth behavior of the amplitude $a(t) = [\ddot{x}](t)$, as will be pointed out later.

<u>Definition</u> 3.1 Let X be a point in $\mathcal{R}$ and $\xi = (X,\eta)$ any process in M. Denote by θ_ξ and T_ξ respectively, the temperature and stress fields, at any time t, at the particle X in the process ξ. Then the constitutive equations of a Maxwellian Non-Conductor are said to be compatible with thermodynamics if the Clausius-Duhem inequality, in the local form

$$(3.1) \qquad -\rho(\dot{\varepsilon}_\xi - \theta_\xi \dot{\eta}) + F^{-1}T_\xi \dot{F} \geq 0$$

is satisfied, whenever $\dot{F}$ and $\dot{\eta}$ exist and are continuous, i.e., at all (X,t) such that $X \neq Y(t)$.

Now, let A and B be any two real numbers and let $\hat{t}\varepsilon(-\infty,t_o)$. If $\xi = (\chi,\eta)$ is a process in M we define a new process $\xi_{\hat{t}} = (\chi_{\hat{t}},\eta_{\hat{t}})$ by

$$(3.2_1) \qquad \chi_{\hat{t}}(X,t) = \begin{cases} \chi(X,t) \;; & -\infty < t < \hat{t} \\ \chi(X,\hat{t}) + (t-\hat{t})AX \;; & \hat{t} \leq t \leq \hat{t} + k \end{cases}$$

$$(3.2_2) \qquad \eta_{\hat{t}}(X,t) = \begin{cases} \eta(X,t) \;; & -\infty < t < \hat{t} \\ \eta(X,\hat{t}) + (t-\hat{t})B \;; & \hat{t} \leq t \leq \hat{t} + k \end{cases}$$

where $k > 0$. We call $\xi_{\hat{t}}$ an $(A,B,\hat{t})$ continuation of ξ. Our additional hypothesis for the class of process M then takes the following form:

(HL) Let $\xi = (\chi,\eta)$ be a process in M. Then for any triple $(A,B,\hat{t})$, where A and B are real numbers and $t\varepsilon(-\infty,t_o)$, there exists a $k > 0$ such that $\xi_{\hat{t}}$, the $(A,B,\hat{t})$ continuation of ξ, is also in M.

<u>Remark 3.1</u> For the deformation gradient $F_{\hat{t}}(X,t)$ we have, obviously

$$(3.3)\qquad F_{\hat{t}}(X,t) = \begin{cases} F(X,t) \ ; \ -\infty < t < \hat{t} \\ F(X,\hat{t}) + (t - \hat{t})A \ ; \ \hat{t} \le t \le \hat{t} + k \end{cases}$$

and the pair $(F_{\hat{t}},\eta_{\hat{t}})$ then represents an $(A,B,\hat{t})$ <u>local linear continuation</u> of (F,η)[2].

Now, let $\xi_{\hat{t}} \in M$ be an $(A,B,\hat{t})$ continuation of $\xi \in M$. Then we make the following additional smoothness assumptions concerning the response of the Maxwellian Non-Conductor under consideration.

(H2) If $\theta_{\xi_{\hat{t}}}$ and $T_{\xi_{\hat{t}}}$ denote, respectively, the stress and temperature fields, at any time $t \in [\hat{t}, \hat{t}+k]$, at the particle X in the process $\xi_{\hat{t}}$ then

$$(3.4_1)\qquad \lim_{t\to\hat{t}} (T_{\xi_{\hat{t}}}(X,t)) = T_{\xi}(X,\hat{t})$$

$$(3.4_2)\qquad \lim_{t\to\hat{t}} (\theta_{\xi_{\hat{t}}}(X,t)) = \theta_{\xi}(X,\hat{t})$$

(H3). If $J_{\xi_{\hat{t}}}$ denotes either $D_{\xi_{\hat{t}}}$, $E_{\xi_{\hat{t}}}$, or $F_{\xi_{\hat{t}}}$, then when $t \in [\hat{t},\hat{t}+k]$,

$$(3.5)\qquad \lim_{t\to\hat{t}} J_{\xi_{\hat{t}}}(F_{\hat{t}}(X,t);\eta_{\hat{t}}(X;t);X;t) = J_{\xi}(F(X,\hat{t});\eta(X;\hat{t});X;\hat{t})$$

Now, as $\xi_{\hat{t}} \in M$, the constitutive equations $(1.2_1) - (1.2_3)$ will be compatible with thermodynamics only if

$$(3.6)\qquad -\rho\dot{\varepsilon}_{\xi_{\hat{t}}} + \rho\theta_{\xi_{\hat{t}}}\dot{\eta}_{\xi_{\hat{t}}} + F_{\hat{t}}^{-1}T_{\xi_{\hat{t}}}\dot{F}_{\hat{t}} \ge 0$$

is satisfied at all points (X,t) where $X \neq Y(t)$. In particular, at

(2) Such continuations have been used by Gurtin [3] and by Wang & Bowen [4].

each $t \in [\hat{t}, \hat{t}+k]$ we must have

$$(3.7) \qquad (F_{\hat{t}}^{-1} T_{\xi_{\hat{t}}}(X,t) - \rho D_{\xi_{\hat{t}}})A + \rho(\theta_{\xi_{\hat{t}}}(X,t) - E_{\xi_{\hat{t}}})B - \rho F_{\xi_{\hat{t}}} \geq 0$$

where we have made use of (3.2_2), (3.3), and (1.2_1) - (1.2_3) with $\xi = \xi_{\hat{t}}$. In (3.7) the arguments of $D_{\xi_{\hat{t}}}$, $E_{\xi_{\hat{t}}}$, and $F_{\xi_{\hat{t}}}$ are $F_{\hat{t}}(X,t)$, $\eta_{\hat{t}}(X,t)$, X, and t. Since (3.7) is valid for each $t \in [\hat{t}, \hat{t}+k]$ if we let $t \to \hat{t}$ then, by virtue of (3.3) and the smoothness hypotheses (H1), (H2), and (H3) if follows that

$$\begin{aligned}(3.8) \qquad & (F^{-1}(X,\hat{t})T_\xi(X,\hat{t}) - \rho D_\xi(F(X,\hat{t});\eta(X,\hat{t});X;\hat{t}))A \\ & + \rho(\theta_\xi(X,\hat{t}) - E_\xi(F(X,\hat{t});\eta(X,\hat{t}),X;\hat{t}))B \\ & - \rho F_\xi(F(X;\hat{t});\eta(X,\hat{t});X;\hat{t}) \geq 0 .\end{aligned}$$

However, as A and B were taken to be any arbitrary real numbers, and $\hat{t}$ is any time in the interval $(-\infty, t_o)$, it follows from (3.8) that in any process $\xi \in \mathcal{M}$

$$(3.9_1) \qquad T_\xi(X,t) = \rho F(X,t) D_\xi(F(X,t);\eta(X;t);X;t)$$

$$(3.9_2) \qquad \theta_\xi(X,t) = E_\xi(F(X;t);\eta(X;t);X;t)$$

$$(3.9_3) \qquad - \rho F_\xi(X;t);\eta(X,t);X;t) \geq 0 - e\ F_\xi(F(X;t);\eta(X,t);X;t)$$

at each $(X,t) \in \mathcal{R} \times (-\infty, t_o)$.

<u>Remark 3.2</u> At each (X,t), where $X \neq Y(t)$, (3.9_1) & (3.9_2) imply that

$$(3.10_2) \qquad \dot{T}_\xi(X,t) = \{\rho F(\partial_F D_\xi)\}\dot{F}(X,t) + \{\rho F(\partial_\eta D_\xi)\}\dot{\eta}(X,t) + \rho F \partial_t D_\xi$$

$$(3.10_2) \qquad \dot{\theta}_\xi(X,t) = \{\partial_F E_\xi\}\dot{F}(X,t) + \{\partial_\eta E_\xi\}\dot{\eta}(X,t) + \partial_t E_\xi$$

These representations for $\dot{T}_\xi$ and $\dot{\theta}_\xi$ are, however, distinct from those assumed in (1.2_1) and (1.2_3) as the terms $\rho F \partial_t D_\xi$, $\partial_t E_\xi$ and the various coefficients of $\dot{F}$ and $\dot{\eta}$ in the above equations are, at most, continuous in the arguments F, η, X, and t.

4. Wave and Entropic Amplitudes

We now turn our attention to a study of the behavior of acceleration waves in Maxwellian Non-Conductors of class 1. As a first step in this direction we show that every such wave must be homentropic, i.e., that it satisfies $[\dot{\eta}] = [\partial_X \eta] = 0$. We assume that the conditions (H1), (H2) and (H3) are satisfied so that the reprsentations (3.9_1) and (3.9_2) are valid for each $\xi \in M$. It then follows that for each $\xi \in M$, $T_\xi(X,t)$ is a continuous function of X for all $X \in R$. So, by (2.6_2) we have

$$(4.1) \qquad \rho_R[\dot{\varepsilon}] = T(Y(t),t)[\dot{F}]$$

However, if we take the jump of (1.2_2) we find that

$$(4.2) \qquad [\dot{\varepsilon}] = (D_\xi)_t[\dot{F}] + (E_\xi)_t[\dot{\eta}] ,$$

where $(D_\xi)_t = D_\xi(F(Y(t),t);\eta(Y(t),t);Y(t);t)$ and $(E_\xi)_t = E_\xi(F(Y(t),t);\eta(Y(t),t);Y(t);t)$. But, by (3.9_1) it is clear that $T_\xi(Y(t),t)/\rho_R = D_\xi(F(Y(t),t);\eta(Y(t),t);Y(t);t)$ as $\rho_R = \rho(Y(t))F(Y(t),t)$. Thus combining (4.1) with (4.2) (and making use of (3.9_2)) yields

$$(4.3)\qquad \theta_\xi(Y(t),t)[\dot{\eta}] = 0$$

If, as is customary, we assume that $\theta_\xi > 0$ in every process $\xi \in M$, then (4.3) clearly implies that $[\dot{\eta}] = 0$ and the result follows from the kinematical condition of compatability

$$(4.4)\qquad [\dot{\eta}] = -U_t[\partial_X\eta] .$$

By making use of the fact that every acceleration wave is necessarily homentropic, we easily establish a formula for the intrinsic velocity U_t of such a wave; taking the jump of (1.2_1) yields

$$(4.5)\qquad [\dot{T}] = (A_\xi)_t[\dot{F}]$$

where $(A_\xi)_t = A_\xi(F(Y(t),t);\eta(Y(t),t);Y(t);t)$. However, as T_ξ is continuous, Maxwell's theorem tells us that

$$(4.6)\qquad [\dot{T}] = -U_t[\partial_X T]$$

and if we combine this result with (2.6_1) and (2.3_1) we easily find

$$(4.7)\qquad [\dot{T}] = -U_t\rho_R[\ddot{x}] = e_\rho U_t^2[\dot{F}]$$

Then, a comparison of (4.5) and (4.7) yields:

<u>Theorem 4.1</u> The instrinsic velocity U_t of an acceleration wave in a Maxwellian Non-Conductor of class 1 is given by

$$(4.8)\qquad U_t^2 = (A_\xi)_t/\rho_R$$

where $(A_\xi)_t = A_\xi(F(Y(t),t);\eta(Y(t);t);Y(t);t)$.

In order to consider the growth behavior of acceleration waves in Maxwellian Non-Conductors we now make use of the equation ([5])

$$(4.9) \qquad 2\,\frac{da}{dt} = \frac{a}{U_t}\,\frac{dU_t}{dt} + \frac{1}{\rho_R}\left[\frac{\partial^2 F}{\partial t \partial X}\right] - U_t^2\left[\frac{\partial \dot{F}}{\partial X}\right]$$

whose derivation is based only on the conditions which define an acceleration wave, our smoothness assumptions on the extrinsic body force, and the law of balance of momentum.

Remark 4.1 We will use our constitutive equations to calculate $[\partial_X \partial_t T_\xi]$. As in Coleman, Greenberg, and Gurtin [5], we may then replace $[\partial_t \partial_X T_\xi]$ by $[\partial_X \partial_t T_\xi]$.

Now, at all points $(X,t) \neq (Y(t),t)$, we differentiate (1.2_1) thru w.r.t X so as to obtain

$$(4.10) \qquad \partial_X \partial_t T_\xi = A_\xi \partial_X \dot{F} + B_\xi \partial_X \dot{\eta} + \{(\partial_F A_\xi)\partial_X F + (\partial_\eta A_\xi)\partial_X \eta + \partial_X A_\xi\}\dot{F} + \{(\partial_F B_\xi)\partial_X F + (\partial_\eta B_\xi)\partial_X \eta + \partial_X B_\xi\}\dot{\eta} + (\partial_F C_\xi)\partial_X F + (\partial_\eta C_\xi)\partial_X \eta + \partial_X C_\xi$$

Taking the jump of (4.10) and making use of our smoothness assumptions on A_ξ, B_ξ, and C_ξ, the fact that the wave is homentropic, and theorem 4.1, gives us

$$(4.11) \qquad [\partial_X \partial_t T_\xi] - \rho_R U_t^2[\partial_X \dot{F}] = (B_\xi)_t[\partial_X \dot{\eta}] + (\partial_F A_\xi)_t[\dot{F}][\partial_X F] + \{(\partial_F A_\xi)_t(\partial_X F)^+ + (\partial_\eta A_\xi)_t(\partial_X \eta)^+ + (\partial_X A_\xi)_t\}[\dot{F}] + \{(\partial_F A_\xi)_t \dot{F}^+ + (\partial_F B_\xi)_t \dot{\eta}^+ + (\partial_F C_\xi)_t\}[\partial_X F] ,$$

where the t subscripts on the quantities enclosed in parentheses indicate that they are to be evaluated at the wave, i.e., at $(Y(t),t)$. Now, as the wave is homentropic it follows that

$$(4.12) \qquad 0 = \frac{d}{dt}[\dot{\eta}] = [\ddot{\eta}] + U_t[\partial_X \dot{\eta}]$$

The quantity $[\dot{\eta}]$ is called the <u>entropic amplitude</u>. If we set

$$(4.13) \qquad \nu_t \equiv (\partial_F A_\xi)_t (\partial_X F)^+ + (\partial_\eta A_\xi)_t (\partial_X \eta)^+ + (\partial_X A_\xi)_t$$

$$(4.14) \qquad \mu_t \equiv (\partial_F A_\xi)_t \dot{F}^+ + (\partial_F B_\xi)_t \dot{\eta}^+ + (\partial_F C_\xi)_t$$

then use of (4.13), (2.3_1), and (2.3_2) enables us to rewrite (4.12) in the form

$$(4.15) \qquad [\partial_X \partial_t T_\xi] - \rho_R U_t^2 [\partial_X \dot{F}] = \frac{-(B_\xi)_t}{U_t}[\ddot{\eta}] - \frac{(\partial_F A_\xi)_t}{U_t^3} a^2(t)$$

$$+ \left\{ \frac{\mu_t}{U_t^2} - \frac{\nu_t}{U_t} \right\} a(t)$$

In order to proceed further, we must relate the entropic amplitude to the amplitude $a(t)$ of the wave. To this end we differentiate (2.5_2) thru w.r.t. X and find that

$$(4.16) \qquad \rho_R \partial_X \dot{\varepsilon} = T_\xi \partial_X \dot{F} + \dot{F} \partial_X T_\xi + \rho_R \partial_X r$$

$$= T_\xi \partial_X \dot{F} + \rho_R \dot{F} \ddot{x} - \rho_R \dot{F} b + \rho_R \partial_X r$$

where use has been made of (2.5_1); we remark that all the terms on the right hand side of (4.16_1) exist and are continuous, at all points

away from the wave, by virtue of (3.9_1), our smoothness assumptions on D_ξ and r, and the conditions which define an acceleration wave. Thus, taking the jump of (4.16_2) yields

$$(4.17) \qquad [\partial_X \dot{\varepsilon}] = \frac{1}{\rho_R} T_\xi(Y(t),t)[\partial_X \dot{F}] + [\ddot{x}\dot{F}] - b(Y(t))[\dot{F}]$$

On the other hand, if we differentiate our constitutive equation (1.2_2) for $\dot{\varepsilon}$ thru w.r.t. X we obtain

$$(4.18) \qquad \partial_X \dot{\varepsilon} = D_\xi \partial_X \dot{F} + E_\xi \partial_X \dot{\eta} + \dot{F}\{(\partial_F D_\xi)\partial_X F + (\partial_\eta D_\xi)\partial_X \eta + \partial_X D_\xi\}$$
$$+ \dot{\eta}\{(\partial_F E_\xi)\partial_X F + (\partial_\eta E_\xi)\partial_X \eta + \partial_X D_\xi\}$$
$$+ (\partial_F F_\xi)\partial_X F + (\partial_\eta F_\xi)\partial_X \eta + \partial_X F_\xi$$

Once again, all the quantities appearing on the right-hand side of this equation exist and are continuous at all points away from the wave and so taking the jump of (4.18) yields

$$(4.19) \qquad [\partial_X \dot{\varepsilon}] = (D_\xi)_t[\partial_X \dot{F}] + (E_\xi)_t[\partial_X \dot{\eta}] + (\partial_F D_\xi)_t[\dot{F}][\partial_X F]$$
$$+ \{(\partial_F D_\xi)_t(\partial_X F)^+ + (\partial_\eta D_\xi)_t(\partial_X \eta)^+ + (\partial_X D_\xi)_t\}[\dot{F}]$$
$$+ \{(\partial_F D_\xi)_t \dot{F}^+ + (\partial_F E_\xi)_t \dot{\eta}^+ + (\partial_F F_\xi)_t\} [\partial_X F]$$

where, as before, the t subscripts indicate that the quantities in the parentheses are to be evaluated at the wave. Now set

$$(4.20_1) \qquad \gamma_t = (\partial_F D_\xi)_t(\partial_X F)^+ + (\partial_\eta D_\xi)_t(\partial_X \eta)^+ + (\partial_X D_\xi)_t$$

$$(4.20_2) \qquad \delta_t = (\partial_F D_\xi)_t \dot{F}^+ + (\partial_F E_\xi)_t \dot{\eta}^+ + (\partial_F F_\xi)_t$$

Since $\frac{1}{\rho} T_\xi(Y(t),t) = (D_\xi)_t$ and $(E_\xi)_t = \theta_\xi(Y(t),t)$ a comparison of (4.17) with (4.19) yields the result:

$$(4.21) \qquad \theta_\xi(Y(t),t)[\partial_X\dot{\eta}](t) = [\ddot{x}\dot{F}] - (\partial_F D_\xi)_t[\dot{F}][\partial_X F] - (\gamma_t + b(Y(t))[\dot{F}] - \delta_t[\partial_X F]$$

Since,

$$(4.22) \qquad [\ddot{x}\dot{F}](t) = a(t)\dot{F}^{+} + (\ddot{x})^{+}[\dot{F}] + a(t)[\dot{F}] = a(t)\{\dot{F}^{+} - (\ddot{x})^{+}/U_t\} - 1/U_t\, a^2(t)$$

if we make use of (2.3), we may rewrite (4.21) in the form

$$(4.23) \qquad \theta_\xi(Y(t),t)[\partial_X\dot{\eta}](t) = \{(\partial_F D_\xi)_t/U_t^3 - 1/U_t\}a^2(t) + H_t a(t)$$

where we have set

$$(4.24) \qquad H_t \stackrel{\text{def}}{=} \dot{F}^{+} - (\ddot{x})^{+}/U_t + \gamma_t/U_t + b(Y(t))/U_t - \delta_t/U_t^2$$

Finally, (4.12) leads us to the following

Theorem 4.2 The entropic amplitude of an acceleration wave which is propagating in a Maxwellian Non-Conductor of class 1 is related to the amplitude a(t) of the wave by

$$(4.25) \qquad [\ddot{\eta}](t) = \frac{1}{\theta_\xi(Y(t),t)}\Big(\{1 - (\partial_F D_\xi)_t/U_t^2\}a^2(t) - H_t U_t a(t)\Big)$$

Remark 4.2 As (4.25) shows, in a general Maxwellian Non-Conductor of class 1 the entropic amplitude is not proportional to the amplitude of the wave. In order for the two amplitudes to be proportional it is necessary and sufficient that

$$(4.26) \qquad (\partial_F D_\xi)_t = U_t^2 = (A_\xi)_t/\rho_R$$

Now, by (3.10_1), it is clear that a representation of the form

$$(4.27) \qquad \dot{T}_\xi = (\rho_R \partial_F D_\xi)\dot{F} + (\rho_R \partial_\eta D_\xi)\dot{\eta} + \rho_R \partial_t D_\xi$$

exists for $\dot{T}_\xi$, at all $X \neq Y(t)$, but, as we have already pointed out, this representation is, in general, distinct from the one given by the constitutive assumption (1.2_1); a conclusion such as (4.26) is, therefore, not possible in the general case. In a simple non-conductor with fading memory, which is sufficiently smooth enough to admit a representation as a Maxwellian Non-Conductor of class 1, the condition expressed by (4.26) is fulfilled as is demonstrated in the appendix.

If we now substitute the expression for the entropic amplitude, as given by (4.25), into (4.15) and make use of (3.9_2), we find that

$$(4.28) \qquad \frac{1}{\rho_R}[\partial_X \partial_t T_\xi] - U_t^2[\partial_X \dot{F}] = \alpha_t a(t) + \beta_t a^2(t)$$

where

$$(4.29) \qquad \rho_R \alpha_t \overset{\text{def}}{=} (B_\xi)_t H_t/(E_\xi)_t - \mu_t/U_t^2 - \nu_t/U_t$$

$$(4.30) \qquad \rho_R \beta_t \overset{\text{def}}{=} \{(B_\xi)_t(\partial_F D_\xi)_t - (E_\xi)_t(\partial_F A_\xi)_t/(E_\xi)_t U_t^3\} - (B_\xi)_t/(E_\xi)_t U_t$$

Finally, so as to be able to employ (4.10), we calculate an expression for dU_t/dt. Differentiating (4.8) w.r.t. t yields the following sequence of results:

$$(4.31)\qquad 2\rho_R U_t \frac{dU_t}{dt} = \frac{d}{dt}\Big(A_\xi(F(Y(t),t);\ \eta(Y(t);t);Y(t);t)\Big)$$

$$= \lim_{X\ Y(t)^+} \{(\partial_F A_\xi)((\partial_X F)\frac{dX}{dt} + (\partial_t F))$$

$$+ (\partial_\eta A_\xi)((\partial_X \eta)\frac{dX}{dt} + (\partial_t \eta)) + (\partial_X A_\xi)\frac{dX}{dt} + (\partial_t A_\xi)\}$$

$$= (\partial_F A_\xi)_t\ \{(\partial_X F)^+ U_t + (\partial_t F)^+\} + (\partial_\eta A_\xi)_t\{(\partial_X \eta)^+ U_t + (\partial_t \eta)^+\}$$

$$+ (\partial_X A_\xi)_t\ U_t + (\partial_t A_\xi)_t$$

If we divide both sides of (4.31$_3$) thru by $2\rho_R U_t^2$ and make use of (4.2) we find that (3)

$$(4.32)\qquad \frac{1}{U_t}\frac{dU_t}{dt} = (\partial_F K_\xi)_t\ \{(\partial_X F)^+ U_t + (\partial_t F)^+\}$$

$$+ (\partial_\eta K_\xi)_t\ \{(\partial_X \eta)^+ U_t + (\partial_t \eta)^+\} + (\partial_X K_\xi)_t\ U_t + (\partial_t K_\xi)_t$$

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(3) If ρ_R is not constant over R there appears on the left hand side of this equation the extra term $(\partial_X \rho_R^t)\ U_t/2\rho_R^t$.

where we have set $K_\xi \overset{\text{def}}{=} \ell n\, A_\xi^{1/2}$. Combining (4.10), (4.28), and (4.32) we are led to the following result:

Theorem 4.3 The growth behavior of the amplitude of an acceleration wave which is propagating with intrinsic velocity U_t in a Maxwellian Non-Conductor of Class 1 is governed by the differential equation

$$(4.33) \qquad 2\,\frac{da(t)}{dt} = \sigma_t a(t) + \beta_t a^2(t)$$

where, $\sigma_t \overset{\text{def}}{=} \frac{1}{U_t}\frac{dU_t}{dt} + \alpha_t$, is determined by (4.32) and (4.29) and β_t is defined by (4.30) .

Equation (4.33) is, a differential equation of Bernoulli type; in general, the coefficients σ_t and β_t will be rather complicated functions which involve, not only the values of various constitutive quantities and their derivatives at the wave, but also the values of both the spatial and time derivatives of the deformation gradient and the entropy just ahead of the wave. Certain simplifications occur, however, if we assume that the wave is propagating into a homogeneous rest region of the body, which we define as follows: Let $\xi = (\chi,\eta)$ be a process in M and let $\sum = \{(X,t) \mid X = Y(t), -\infty<t<t_o\}$ denote an acceleration wave relative to ξ which is propagating in the body with intrinsic velocity U_t. Then the wave is said to be propagating into a homogeneous rest region in the time interval $[0, t_o)$ if (i) for $0 \le t < t_o$ and $X \ge Y(t)$, $F(X,t) = F_o$=const. and $\eta(X,t) = \eta_o$=const. (ii) $A_\xi, B_\xi,\ldots,I_\xi$ are all constant in X and t for $0 \le t < t_o$ and $X \ge Y(t)$.

Thus $\dot{F}^+ = (\partial_X F)^+ = \dot{\eta}^+ = (\partial_X \eta)^+ = 0$ and, for $0 \le t < t_o$ and $X \ge Y(t)$,

$A_\xi(F(X,t);\ \eta(X;t);\ X;t) = A_\xi(F_o,\ \eta_o,\ X,\ t) = A_\xi^o, \ldots,$

$I_\xi(F(X,t);\eta(X,t);X;t) = I_\xi(F_o;\eta_o;X;t) = I_\xi^o$.

(iii) $\rho_R(X) = \rho_o$ = const., for $X \ge Y(t)$ {in the case where ρ_R is not already assumed constant over all of R }.

It is then clear that $U_t^2 = \dfrac{A_\xi^o}{\rho_o} = U_o^2$ = const., $0 \le t < t_o$, and

$$\nu_t = (\partial_X A_\xi)_t = \lim_{X \to Y(t)^+} \partial_X A_\xi(F_o;\ \eta_o;\ X;t) = 0 \tag{4.34}$$

$$\mu_t = (\partial_F C_\xi)_t = \lim_{X \to Y(t)^+} \partial_F C_\xi(F_o;\ \eta_o;\ X;t) = (\partial_F C_\xi)_o = \text{const.} \tag{4.35}$$

$$\gamma_t = (\partial_X D_\xi)_t = \lim_{X \to Y(t)^+} \partial_X D_\xi(F_o;\ \eta_o;X;t) = 0 \tag{4.36}$$

$$\delta_t = (\partial_F F_\xi)_t = \lim_{X \to Y(t)^+} \partial_F F_\xi(F_o;\ \eta_o;X;t) = (\partial_F F_\xi)_o = \text{const.} \tag{4.37}$$

$$H_t = b(Y(t))/U_o - (\partial_F F_\xi)_o/U_o^2 \tag{4.38}$$

However, by (2.5_1), (3.9_1), and our smoothness assumptions on b,

$$\begin{aligned} \rho_R b\ (Y(t)) &= \rho_R \ddot{x}^+ - (\partial_X T)^+ \\ &= \rho_R \{\ddot{x}^+ - (\partial_F D_\xi)_t\ (\partial_X F)^+ - (\partial_\eta D_\xi)_t\ (\partial_X \eta)^+ \\ &\qquad - (\partial_X D_\xi)_t - (\partial_t D_\xi)_t\} = 0, \end{aligned} \tag{4.39}$$

as all the terms on the right hand side of this equation vanish. It then follows that

$$(4.40) \qquad H_t = -(\partial_F F_\xi)_o/U_o^2 \equiv H_o = \text{const.}$$

In view of (4.34) - (4.38) and (4.40), (4.29) and (4.30) now assume the relatively simple forms

$$(4.41) \qquad \alpha_t = \frac{1}{\rho_o}\{B_\xi^o H_o/E_\xi^o - (\partial_F C_\xi)_o/U_o^2\} = \alpha_o = \text{const.}$$

$$(4.42) \qquad \beta_t = \frac{1}{\rho_o}\{[B_\xi^o(\partial_F D_\xi)_o - E^o(\partial_F A_\xi)_o]/E_\xi^o U_o^3 - B_\xi^o/E_\xi^o U_o\} = \beta_o = \text{cons}$$

Corollary: The growth behavior of the amplitude of an acceleration wave which is propagating into a homogeneous rest region of a Maxwellian Non-Conductor of class 1, in the time interval $[0,t_o)$, is governed by the differential equation

$$(4.43) \qquad 2\,\frac{da(t)}{dt} = \alpha_o a(t) + \beta_o\, a^2(t) \;;\; 0 \le t < t_o$$

where α_o and β_o are the constants given by (4.41) and (4.42).

5. Acceleration Waves in Maxwellian Non-Conductors: Growth and Decay Behavior

The solutions of the differential equations (4.33) and (4.43) are well-known as these equations and various special cases of them

have arisen frequently in studies on the growth behavior of acceleration waves in non-linear materials of various kinds, i.e. see [2] [5], or [6]; we record here the appropriate results as they apply to acceleration waves in Maxwellian Non-Conductors of class 1. Consider the differential equation (4.33) and assume that $U_t > 0$, $0 \le t < t_o$. It is easily verified that, by virtue of our smoothness assumptions on the constitutive quantities $A, \ldots I$ and the conditions which define an acceleration wave, the coefficients σ_t and β_t are continuous functions of t for $0 \le t < t_o$. If $a(0) \stackrel{\text{def}}{=} \lim_{t \to 0^+} a(t)$ exists, then it is a consequence of the uniqueness of solutions of (4.33) that $a(t^*) = 0$ at some time $t^* \in [0,t_o)$ implies that $a(t) \equiv 0$ for all $t \in [0,t_o)$. So, assume that $a(t) \neq 0$, $0 \le t < t_o$. We can then state

<u>Theorem 5.1</u> The amplitude $a(t)$ of an acceleration wave which is propagating in a Maxwellian non-conductor of class 1 in the time interval $[0,t_o)$ is given by

$$(5.1) \qquad a(t) = \frac{a(o)\, e^{-\psi(t)}}{1 - \frac{a(o)}{2} \int_o^t \beta_\tau e^{-\psi(\tau)} d\tau}, \quad 0 \le t < t_o ;$$

here β_t is defined by (4.30), $\psi(t) = -1/2 \int_o^t \sigma_\tau d\tau$ and $\sigma_t = \frac{1}{U_t} \frac{dU_t}{dt} + \alpha_t$ where U_t (the intrinsic velocity) satisfies (4.32) and α_t is defined by (4.29).

Now rewrite (5.1) in the form

$$(5.2) \qquad \frac{a(t)}{a(0)} = \frac{e^{-\psi(t)}}{1 + a(o)I(t)}$$

where

$$(5.3) \qquad I(t) = -\,{}^1\!/_2 \int_o^t \beta_\tau e^{-\psi(\tau)} d\tau .$$

In the general case we can say very little about the growth behavior of the amplitude $a(t)$. If, however, the Maxwellian Non-Conductor is such as to satisfy (4.26) then (4.30) clearly reduces

to $\quad \beta_t = -\,(\partial_F A_\xi)_t \,/\, (A_\xi)_t\, U_t$

and if we set $\pi_t = -\,{}^1\!/_2\ \beta_t = (\partial_F A_\xi)_t \,/\, 2(A_\xi)_t\, U_t$. (5.3) becomes

$$(5.4) \qquad I(t) = \int_o^t \pi_\tau\, e^{-\psi(\tau)} d\tau$$

Clearly $I(0) = 0$. Moreover, $\operatorname{sgn} I(t) = \operatorname{sgn} (\partial_F A_\xi)_t$ [4], and $I(t)$ is strictly monotone for $0 \le t < t_o$, provided $(\partial_F A_\xi)_t \ne 0$, $0 \le t < t_o$; this last statement follows from the continuity of $\partial_F A_\xi$ which guarantees that $(\partial_F A_\xi)_t$ is of fixed sign for $0 \le t < t_o$ if

$(\partial_F A_\xi)_t \ne 0$ on $[0, t_o)$. Finally we assume that the material and the process just ahead of the wave are well-behaved in the sense that $(\partial_X F)^+(t)$, $(\dot{F}^+)(t)$, $(\partial_X \eta)^+(t)$, $(\dot{\eta}^+)(t)$, $(\partial_F A_\xi)_t$, $(\partial_X A_\xi)_t$ $(\partial_X A_\xi)_t$, $(\partial_t A_\xi)_t$, $(B_\xi)_t$, $(\partial_F B_\xi)_t$, $(\partial_F C_\xi)_t$, $(\partial_F D_\xi)_t$, $(\partial_\eta D_\xi)_t$, $(\partial_X D_\xi)_t$, $(E_\xi)_t$, $(\partial_F E_\xi)_t$, and $(\partial_F F_\xi)_t$ all

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(4) Since we are assuming that $U_t > 0$, $0 \le t < t_o$, we also must have $(A_\xi)_t > 0$, $0 \le t < t_o$.

have finite limits as $t \to t_o$; then, not only are both $\psi(t)$ and $I(t)$ continuous for $0 \leq t < t_o$, but they have finite limits $\psi(t_o)$ and $I(t_o)$, respectively, as $t \to t_o$.

Remark 5.1 When (4.26) is satisfied, our governing equation for the amplitude $a(t)$ is

$$(5.5) \qquad \frac{da}{dt} - \frac{\sigma_t}{2} a + \pi_t a^2 = 0$$

The most important consideration here is the form of the coefficient π_t; If we call $(A_\xi)_t$ the instantaneous tangent modulus at the wave and $(\partial_F A_\xi)_t$ the instantaneous second-order modulus at the wave, then this coefficient has precisely the same form as that of its counterparts in the Bernoulli equations arising, for example, in the studies [2], [5] and [6]. A broad analysis of the local and global behavior of acceleration waves whose amplitudes obey Bernoulli's equations of the form (5.5), in which π_t, the coefficient of the term $a^2(t)$, has the form

$$(5.6) \qquad \pi_t = E_t/2\tilde{E}_t U_t$$

(with E_t and $\tilde{E}_t$ being, respectively, the appropriately defined tangent and second-order tangent moduli, at the wave, for the particular material under consideration) has been carried out by Bailey & Chen in [7] and [8]. In all instances where Bernoulli equations of this form appear, in studies on the growth behavior of waves propagating in non-linear materials, the intrinsic velocity U_t satisfies the relation

(5.7) $$U_t^2 = E_t/\rho_R^t$$

where $\rho_R^t = e_R(X)|_{X = Y(t)}$. The usual assumptions which are then made are that $U_t > 0$, $\rho_R^t > 0$, and $\tilde{E}_t \neq 0$ (the constitutive assumptions, in all cases, guarantee that $\tilde{E}_t$ is a continuous function of t); these, in turn, lead to the conclusion that sgn I(t) = sgn $\tilde{E}_t$ and all subsequent analysis, of the qualitative behavior of the amplitude, a(t), is based on this fact and the continuity of I(t).

Following Coleman, Greenberg, and Gurtin [5] we set

(5.8) $$\lambda(t_o) = -1/I(t_o)$$

Then we have the direct analogue of Remarks 3.4, 3.5, and 3.6 of [5] which we state without proof as

<u>Theorem 5.2</u> Consider a Maxwellian Non-Conductor of Class 1 in which (4.26) is satisfied for every $\xi \in M$. Let ξ be any process in M and Σ an acceleration wave, relative to ξ, which is propagating into the body in the time interval $[0,t_o)$. Then (i) If either $|a(0)| < |\lambda(t_o)|$ or sgn a(0) = sgn $(\partial_F A_\xi)_t$, then a(t) remains bounded throughout the closed interval $[0,t_o]$. (ii) If sgn a(0) = - sgn $(\partial_F A_\xi)_t$ and $|a(0)| > |\lambda(t_o)|$, then there exists a finite time $t_\infty \in (0,t_o)$ such that $\lim_{t\to t_\infty} |a(t)| = \infty$; (this contradicts our assumption that Σ is an acceleration wave relative to ξ throughout $(0,t_o)$). (iii) If sgn a(0) = - sgn $(\partial_F A_\xi)_t$ and $|a(0)| = |\lambda(t_o)|$ then a(t) is continuous for $0 \leq t < t_o$, but $|a(t)| \to \infty$ as $t \to t_o$.

Now consider an acceleration wave which is propagating, in the time interval $[0,t_o)$, into a homogeneous rest region of a Maxwellian Non-Conductor of class 1; the governing equation for the amplitude is, in this case, (4.43) where α_o and β_o are constants which are defined, respectively, by (4.41) and (4.42). We rewrite this equation in the form

$$\frac{da}{dt} + \tilde{\alpha}\, a + \tilde{\beta}\, a^2 = 0 \tag{5.9}$$

where $\tilde{\alpha} = -{}^1/_2\, \alpha_o$ and $\tilde{\beta} = -{}^1/_2 \beta_o$. Then as a direct analogue of Theorem 5.1 of Coleman, Greenberg, and Gurtin [5] we have

<u>Theorem 5.3</u> Consider an acceleration wave Σ which is propagating into a homogeneous rest region of a Maxwellian Non-Conductor of class 1 in the time interval $[0,t_o)$. Then, if α_o and β_o, as defined by (4.41) and (4.42), respectively, are non-zero, the amplitude satisfies

$$a(t) = \frac{\tilde{\kappa}}{\left(\frac{\tilde{\kappa}}{a(0)} - 1\right)e^{\tilde{\alpha} t} + 1}, \quad 0 \le t < t_o \tag{5.10}$$

where

$$\tilde{\kappa} \;\underline{\underline{\text{def}}}\; - \alpha_o/\beta_o \tag{5.11}$$

$$= U_o\{E^o_\xi(\partial_F C_\xi)_o - B^o_\xi H_o U^2_o\}/\; B^o_\xi(\partial_F D_\xi)_o - (\partial_F A_\xi)_o E^o_\xi - B^o_\xi U^2_o$$

$$\tilde{\alpha} = - \frac{1}{2\rho_o}\{B^o_\xi H_o/E^o_\xi - (\partial_F C_\xi)_o/U^2_o\} = \frac{1}{2A^o_\xi}\{B^o_\xi(\partial_F F_\xi)_o/E^o_\xi + (\partial_F C_\xi)_o\}$$

Remark 5.2 The expression (5.10) above is valid only under the assumption that both $\alpha_o \neq 0$ and $\beta_o \neq 0$. If we assume that $\beta_o \neq 0$ but that $(\partial_F C_\xi)_o = 0$ and $B^o = 0$ or $(\partial_F F_\xi) = 0$ then the formula for the amplitude takes the form

$$(5.12) \qquad a(t) = \frac{2a(0)}{2 - \beta_o a(0)t} \quad , \; 0 \leq t < t_o$$

and it is clear that $|a(t)| \to \infty$, whenever sgn β_o = sgn $a(0)$, as $t \to t_\infty = 2/\beta_o a(0)$ provided, of course, that $0 < 2/\beta_o a(0) < t_o$. If either sgn β_o = - sgn $a(0)$ or sgn β_o = sgn $a(0)$ but $2/\beta_o a(0) \geq t_o$ $a(t)$ is continuous for all $t \in [0,t_o)$.

Finally, if we define constants V_o and W_o by

$$(5.13_1) \qquad V_o \stackrel{\text{def}}{=} E^o_\xi \, (\partial_F C_\xi)_o - B^o_\xi \, H_o U_o^2$$

$$(5.13_2) \qquad W_o \stackrel{\text{def}}{=} B^o_\xi \, (\partial_F D_\xi)_o - (\partial_F A_\xi)_o \, E^o_\xi - B^o_\xi \, U_o^2 \quad (5)$$

then we may rewrite $\tilde{\kappa}$ in the form

$$(5.14) \qquad \tilde{\kappa} = U_o V_o / W_o$$

Following, once again, Coleman, Greenberg, and Gurtin [3], let us agree to call the wave weak if $|a(0)| < |\lambda|$ and strong if $|a(0)| > |\lambda|$, where $\lambda = \lambda(t_o)$. Then the assumptions

$$(5.15) \qquad V_o < 0, \; W_o \neq 0, \; U_o > 0, \; \rho_o > 0$$

imply that $a(t) \to 0$, monotonically, as $t \to \infty$ when the wave is weak or when sgn $a(0)$ = sgn W_0. If the wave is strong and sgn $a(0)$ = -sgn W_0 then $|a(t)| \to \infty$ monotonically in finite time.

(5) $W_o = -(\partial_F A_\xi)_o E^o$ if the material satisfies the condition expressed by (4.26)

Appendix:

Simple Non-Conductors as Maxwellian Non-Conductors of Class 1

We sketch here an outline of the proof that every simple non-conductor exhibiting fading memory in the sense of Coleman [1] may be viewed as a Maxwellian non-conductor of class 1 provided certain smoothness asumptions, analogous to those employed in [5], are imposed on the response of the material.

Let F^t and θ^t denote the histories up to time t of the deformation gradient and the temperature respectively, at a fixed material point X; these are real-valued functions defined by

(A.1) $$F^t(s) = F(t-s) = \partial_X \chi(X,t-s);\ 0 \le s < \infty$$

(A.2) $$\theta^t(s) = \theta(t-s) = \theta(X,t-s);\ 0 \le s < \infty$$

Then the constitutive equations of a simple non-conductor are

(A.3) $$T(t) = \mathcal{I}(F^t,\eta^t)$$

(A.4) $$\theta(t) = \hat{\theta}(F^t,\eta^t)$$

(A.5) $$\varepsilon(t) = \hat{\varepsilon}(F^t,\eta^t)$$

(A.6) $$q(t) = \hat{q}(F^t,\eta^t,g) \equiv 0,$$

where $g(t) = \partial_X \theta(X,t)$ is the temperature gradient. We assume that the material is homogeneous and that a homogeneous reference configuration has been chosen so that the response functionals $\mathcal{I},\hat{\theta}$,

and $\hat{\varepsilon}$, are independent of X. Let h be a fixed influence function, i.e., a positive monotone decreasing function in $C^0[0,\infty)$ decaying fast enough to zero so as to be square integrable. Let $\Lambda^t = (F^t, \eta^t)$ be a pair of histories and define the norm $||\Lambda^t||^\theta$ of Λ^t to be

(A.7) $$||\Lambda^t||^\theta = ||F_r^t||_h + ||\eta_r^t||_h + |F^t(0)| + |\eta^t(0)|$$

where F_r^t and η_r^t denote the restrictions of F^t and η^t to the interval $(0,\infty)$ and where

(A.8) $$||f||_h^2 = \int_0^\infty |f(s)|h(s)^2 ds$$

We assume that there exists an influence function h such that $I, \hat{\theta}$, and $\hat{\varepsilon}$, have for their *common* domain of definition an open subset D of the function space $\mathcal{L}$ composed of those function pairs $\Lambda^t = (F^t, \eta^t)$ whose norm $||\Lambda^t||^\theta$ is finite. In addition, we assume that $I, \hat{\theta}$, and $\hat{\varepsilon}$, are C^1 functions over D w.r.t. $||\cdot||^\theta$, i.e. if f denotes either $I, \hat{\theta}$, or $\hat{\varepsilon}$, then f has at each $\Lambda^t \varepsilon D$ a first order Frechet derivative $\delta^\theta f(\Lambda|\cdot)$ which is a continuous linear functional over $\mathcal{L}$ and has the property that for all functions Ω in $\mathcal{L}$ with $\Lambda^t + \Omega$ in D

(A.9) $$f(\Lambda^t + \Omega) = f(\Lambda^t) + \delta^\theta f(\Lambda^t|\Omega) + 0(||\Omega||^{\theta 2})$$

Now, in terms of the past histories, the constitutive equations A.3 - A.5 may be written in the form

(A.10) $$T(t) = I(F_r^t, \eta_r^t; F, \eta)$$

$$(A.11) \qquad \theta = \hat{\theta}(F_r^t;\eta_r^t;F;\eta)$$

$$(A.12) \qquad \varepsilon = \hat{\varepsilon}(F_r^t;\eta_r^t;F,\eta)$$

where $F = F(t)$ and $\eta = \eta(t)$. If f stands for either $I, \hat{\theta}$, or $\hat{\varepsilon}$, in A.10 - A.12, then the smoothness assumption (A.9) (fading memory) implies the existence of differential operators $D_F, D_\eta, \delta_F, \delta_\eta$ which operate on f to yield functionals $D_F f, D_\eta f, \delta_F f, \delta_\eta f$ as follows: Let $\Lambda^t = (F^t,\eta^t)\varepsilon D$ and $g\varepsilon H_h$ where H_h is the Hilbert space of all real-valued functions on $[0,\infty)$ satisfying $||g||_h < \infty$. Then,

$$(A.13) \qquad D_F f(F^t;\eta^t) = \frac{\partial}{\partial F} f(F_r^t;\eta_r^t;F,\eta)$$

$$(A.14) \qquad D_\eta f(F^t;\eta^t) = \frac{\partial}{\partial \eta} f(F_r^t;\eta_r^t;F;\eta)$$

$$(A.15) \qquad \delta_F f(F^t;\eta^t|g) = \frac{\partial}{\partial \nu} f(F_r^t + \nu g,\eta_r^t;F;\eta)|_{\nu=0}$$

$$(A.16) \qquad \delta_\eta f(F^t;\eta^t|g) = \frac{\partial}{\partial \nu} f(F_r^t;\eta_r^t + \nu g;F;\eta)|_{\nu=0}$$

where the partial Frechet derivatives $\delta_F f$ and $\delta_\eta f$ are jointly continuous in all their arguments including g, and are linear in g. By the Reisz representation theorem this implies, in particular, for $f = I$, the existence of functions K_t and L_t in H_h satisfying

$$(A.17) \qquad \delta_F f(F^t,\eta^t|g) = \int_0^\infty g(s)K_t(s)h(s)^2 ds$$

$$(A.18) \qquad \delta_\eta f(F^t,\eta^t|g) = \int_0^\infty g(s)L_t(s)h(s)^2 ds$$

Now let $\hat{K}_t$ and $\hat{L}_t$ be the unique solutions, respectively, of the systems

(A.19) $$\frac{d}{ds} \hat{K}_t(s) = \hat{K}_t(s)h(s)^2; \hat{K}_t(0) = D_F I(F^t, \eta^t)$$

(A.20) $$\frac{d}{ds} \hat{L}_t(s) = \hat{L}_t(s)h(s)^2; \hat{L}_t(0) = D_\eta I(F^t, \eta^t)$$

Clearly $\hat{K}_t$ and $\hat{L}_t$ will depend on the histories F^t and η^t, i.e. we may write

(A.21) $$\hat{K}_t(s) = K(F^t;\eta^t;s)$$

(A.22) $$\hat{L}_t(s) = L(F^t;\eta^t;s)$$

The functions $\hat{K}_t$ and $\hat{L}_t$ are called, respectively, the stress-strain and stress-entropy relaxation functions for the material. We assume that the following smoothness assumptions are satisfied: For each fixed s, $K(\cdot,\cdot,s)$ and $L(\cdot,\cdot,s)$ are continuous functionals over D. For each fixed pair $(F^t,\eta^t)\varepsilon D$, $K'(F^t;\eta^t;s)$ and $L'(F^t;\eta;s)$ are differentiable functions of s, i.e., for $0 \le s < \infty$

(A.23) $$L''(F^t;\eta^t;s) = \frac{\partial}{\partial s} K'(F^t;\eta^t;s)$$

(A.24) $$L''(F^t;\eta^t;s) = \frac{\partial}{\partial s} L'(F^t;\eta^t;s)$$

exist and, moreover, satisfy

(A.25) $\quad K''(F^t;\eta^t;\cdot)h(\cdot)^2 \epsilon \mathcal{H}_h$

(A.26) $\quad L''(F^t;\eta^t;\cdot)h(\cdot)^2 \epsilon \mathcal{H}_h$

Finally, we assume that for each pair $(F^t;\eta^t)\epsilon D$

(A.27) $\quad K'(F^t;\eta^t;\cdot)h(\cdot)^2 \epsilon \mathcal{H}_h$

(A.28) $\quad L'(F^t;\eta^t;\cdot)h(\cdot)^2 \epsilon \mathcal{H}_h$

Clearly, we may rewrite (A.17) and (A.18) in the forms

(A.29) $\quad \delta_F I(F^t;\eta^t|g) = \int_0^\infty g(s)K'(F^t;\eta^t;s)ds$

(A.30) $\quad \delta_\eta I(F^t;\eta^t|g) = \int_0^\infty g(s)L'(F^t;\eta^t;s)ds$

In a similar fashion, if we take $f=\hat{\varepsilon}$, we find that

(A.31) $\quad \delta_F \hat{\varepsilon}(F^t;\eta^t\ g) = \int_0^\infty g(s)M'(F^t;\eta^t;s)ds$

(A.32) $\quad \delta_\eta \hat{\varepsilon}(F^t;\eta^t|g) = \int_0^\infty g(s)N'(F^t;\eta^t;s)ds$

where $\hat{M}_t(s) = M(F^t;\eta^t;s)$ and $\hat{N}_t(s) = N(F^t;\eta^t;s)$ are, respectively, the energy-strain and energy-entropy relaxation functions for the material; they satisfy $M(F^t;\eta^t,0) = D_F\hat{\varepsilon}(F^t;\eta^t)$, $N(F^t;\eta^t;0) = D_\eta\hat{\varepsilon}(F^t;\eta^t)$ as well as all the smoothness conditions laid down for $K(F^t;\eta^t;s)$ and $L(F^t;\eta^t;s)$. Finally

(A.33) $\quad \delta_F\hat{\theta}(F^t;\eta^t|g) = \int_0^\infty g(s)P'(F^t,\eta^t;s)ds$

(A.34) $$\delta_\eta \hat{\theta}(F^t,\eta^t|g) = \int_0^\infty g(s)R'(F^t;\eta^t;s)ds$$

where $\hat{P}_t(s) = P(F^t;\eta^t;s)$ and $\hat{R}_t(s) = R(F^t;\eta^t;s)$ are, respectively, the temperature-strain and temperature-entropy relaxation functions, and $\hat{P}(F^t;\eta^t;0) = D_F\theta(F^t;\eta^t)$, $\hat{R}(F^t;\eta^t;0) = D_\eta\theta(F^t;\eta^t)$ and once again, the smoothness conditions laid down for $K(F^t;\eta^t;s)$ and $L(F^t;\eta^t;s)$ are assumed to be satisfied by $P(F^t;\eta^t;s)$ and $R(F^t;\eta^t;s)$.

Now, our smoothness assumption (A.9), where f stands for either $I,\hat{\theta}$, or $\hat{\varepsilon}$ allows us to differentiate each of these functionals w.r.t. time so as to obtain

(A.35) $$\begin{aligned}\dot{T} &= D_F I(F^t;\eta^t)\dot{F} + D_\eta I(F^t\cdot\eta^t)\dot{\eta} \\ &\quad + \delta_F I(F^t;\eta^t|F_r^t) + \delta_\eta I(F^t;\eta^t|\dot{\eta}_r^t) \\ &= K(F^t;\eta^t;0)\dot{F} + L(F^t;\eta^t;0)\dot{\eta} \\ &\quad + \int_0^\infty \{K'(F^t;\eta^t;s)\dot{F}_r^t(s) + L'(F^t;\eta^t;s)\dot{\eta}_r^t(s)\}ds\end{aligned}$$

(A.36) $$\begin{aligned}\dot{\varepsilon} &= D_F\hat{\varepsilon}(F^t;\eta^t)\dot{F} + D_\eta\hat{\varepsilon}(F^t;\eta^t)\dot{\eta} \\ &\quad + \delta_F\hat{\varepsilon}(F^t;\eta^t|\dot{F}_r^t) + \delta_\eta\hat{\varepsilon}(F^t;\eta^t|\dot{\eta}_r^t) \\ &= M(F^t;\eta^t;0)\dot{F} + N(F^t;\eta^t;0)\dot{\eta} \\ &\quad + \int_0^\infty \{M'(F^t;\eta^t;s)\dot{F}_r^t(s) + N'(F^t;\eta^t;s)\dot{\eta}_r^t(s)\}\, ds\end{aligned}$$

(A.37) $$\dot{\theta} = D_F\hat{\theta}(F^t;\eta^t)\dot{F} + D_\eta\hat{\theta}(F^t,\eta^t)\dot{\theta}$$

$$+ \delta_F\hat{\theta}(F^t;\eta^t|\dot{F}^t_r) + \delta_\eta\hat{\theta}(F^t_r;\eta^t_r|\dot{\eta}^t_r)$$

$$= P(F^t;\eta^t;0)\dot{F} + R(F^t;\eta^t;0)\dot{\eta}$$

$$+ \int_0^\infty\{P'(F^t;\eta^t;s)\dot{F}^t_r(s) + R'(F^t;\eta^t;s)\dot{\eta}^t_r(s)\}ds,$$

where each of the above results is clearly valid only at those points (X,t) where both $\dot{F}$ and $\dot{\eta}$ exist. Now let $\mu_X(F^t;\eta^t;s)$ stand for any one of the relaxation functions introduced. We shall assume that the reference configuration $\mathcal{R}$ has been chosen so that each of the following maps from $\mathcal{R} \times D \to R^+$ are of class C^1 w.r.t. the norm $||\cdot||^{\theta}$ (continuously Frechet differentiable):

(A.38) $\qquad (X;F^t;\eta^t) \to \mu_X(F^t;\eta^t;0)$

(A.39) $\qquad (X;F^t;\eta^t) \to \mu'_X(F^t;\eta^t;0)$

(A.40) $\qquad (X;F^t;\eta^t) \to \int_0^\infty\mu''_X(F^t;\eta^t;s)F^t_r(s)ds.$

Finally we assume that D is sufficiently large enough to ensure there exists a non-empty open subset $\mathcal{J}$ of $R^+ \times R^+$ such that, for any fixed $(F^t;\eta^t)\epsilon D$, $\Lambda(\cdot)\epsilon D$ whenever $A \,\epsilon\, \mathcal{J}$ where

(A.41) $$\Lambda(s) = \begin{cases} A \;;\; s=0 \\ (F^t_r(s);\eta^t_r(s)) \;;\; s>0 \end{cases}$$

In particular, this is satisfied if we choose $A=(F^t(0),\eta^t(0))$. We

shall now exhibit a class $\hat{M}$ of pairs (χ,η) where χ is a motion of R and η and entropy density function, which is such that the simple non-conductor defined by (A.3) - (A.6) and satisfying the smoothness conditions above, is a Maxwellian Non-Conductor of class 1 relative to $\hat{M}$. We require, explicity, that each $(\chi,\eta)\varepsilon M$ satisfy the following conditions; (i) χ contains, at most, an acceleration wave of order 2 (ii) η is continuous jointly in X and t while $\dot{\eta}$, $\partial_X\eta, \ddot{\eta}, \partial_X\dot{\eta}$, and $\partial_X{}^2\eta$ have, at most, jump discontinuities across $\sum$ but are continuous in X and t jointly everywhere else. (iii) $\{F^t(X,\cdot),\eta^t(X,\ )\}\varepsilon D$ for each $X\ \varepsilon\ R$ and for all (X,t),$\{F(X,t), \eta(X,t)\}\varepsilon \mathcal{J}$ (iv) the maps, $(X,t)\rightarrow F^t_r(X,\cdot)$, and $(X,t)\rightarrow \eta^t_r(X,\cdot)$, from $Rx(-\infty,t_o)\rightarrow H_h$ are C^1 (continuously Frechet differentiable).

The last hypothesis implies the existence in H_h of the derivatives $\dot{F}^t_r(X,\cdot)$ and $\dot{\eta}^t_r(X,\cdot)$ of, respectively, the maps $t\rightarrow F^t_r(X,\cdot)$ and $t\rightarrow \eta^t_r(X,\cdot)$ from $(-\infty,t_o)$ into H_h. Thus both F^t_r and η^t_r are absolutely continuous and the following relations hold for almost all s in $(0,\infty)$:

(A.42) $$\dot{F}^t_r(X,s) = -\partial_s F^t_r(X,s)$$

(A.43) $$\dot{\eta}^t_r(X,s) = -\partial_s \eta^t_r(X,s)$$

Thus we may rewrite (2.35), (2.36), and (2.37) in the forms

(A.44) $$\dot{T} = K(F^t,\eta^t,0)\dot{F} + L(F^t,\eta^t,0)\dot{\eta} - \int_0^\infty\{K'(F^t,\eta^t,b)\partial_b F^t_r(b) + L'(F^t;\eta;{}^t b)\partial_b\eta^t_r(b)\}db$$

(A.45) $$\dot{\varepsilon} = M(F^t,\eta^t,0)\dot{F} + N(F^t,\eta^t,0)\dot{\eta} - \int_0^\infty\{M'(F^t,\eta^t,b)\partial_b F_r^t(b) + N'(F^t,\eta^t,b)\partial_b \eta_r^t(b)\}db$$

(A.46) $$\dot{\theta} = P(F^t,\eta^t,0)\dot{F} + R(F^t,\eta^t,0)\dot{\eta} - \int_0^\infty\{P'(F^t,\eta^t,b)\partial_b F_r^t(b) R'(F^t,\eta^t,b)\partial_b \eta_r^t(b)\}db,$$

which are valid at all points (X,t) away from the wave $\sum$. Employing integration by parts, in each of the integrals above, we find that the constitutive equations (A.44) - (A.46) may be written in the forms (1.2_1) - (1.2_3) where the functions $A_\xi, B_\xi, \ldots, I_\xi$ are defined as follows:

(A.47) $$\begin{aligned} A_\xi &= K(F^t;\eta^t;0) \\ B_\xi &= L(F^t;\eta^t;0) \\ C_\xi &= K'(F^t;\eta^t;0)F + L'(F^t;\eta^t;0)\eta \\ &\quad + \int_0^\infty\{K''(F^t;\eta^t;b)F_r^t(b) + L''(F^t;\eta^t;b)\eta_r^t(b)\}\,db \end{aligned}$$

(A.48) $$\begin{aligned} C_\xi &= M(F^t;\eta^t;0) \\ E_\xi &= N(F^t;\eta^t;0) \\ F_\xi &= M'(F^t;\eta^t;0)F + N'(F^t;\eta^t;0)\eta \\ &\quad + \int_0^\infty\{M''(F^t;\eta^t;b)F_r^t(b) + N''(F^t;\eta^t;b)\eta_r^t(b)\}db \end{aligned}$$

(A.49) $$\begin{aligned} G_\xi &= P(F^t;\eta^t;0) \\ H_\xi &= R(F^t;\eta^t;0) \\ I_\xi &= P'(F^t;\eta^t;0)F + R'(F^t;\eta^t;0)\eta \\ &\quad + \int_0^\infty\{P''(F^t;\eta^t;b)F_r^t(b) + R''(F^t;\eta^t;b)\eta_r^t(b)\}db \end{aligned}$$

By our smoothness assumptions on the response of the simple non-conductor, and our hypothesis concerning the class $\hat{M}$, each of the functions $A_\xi, B_\xi, \ldots, I_\xi$ above is clearly of class C^1 in the arguments F, η, X, and t.

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